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Effective description of quark mixing

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We use the effective Lagrangian formalism to describe quark mixing. The new $W^{\pm}$, Z and H couplings generalizing the CKM matrix and the GIM mechanism fulfil relations and inequalities which allow to discriminate among different SM extensions. As a by-product we give a useful parametrization of the generalized CKM matrix. We also show that the largest possible departures from the SM predictions result from heavy exotic fermions, which can induce, for example, top FCNC large enough to be observable at future colliders.

1 Introduction

The mixing of fermions with the same quantum numbers provides a very sensitive window to new physics. Two distinguished examples are the prediction of the existence of the charm quark by Glashow, Iliopoulos and Maiani (GIM) [1] from the absence of flavour changing neutral currents (FCNC), and the recent results on neutrino oscillations indicating a non-zero mass for the neutrinos. Here we are interested in the quark sector. The Standard Model (SM) predicts a definite pattern of quark mixing: FCNC are absent at tree level and suppressed at one loop, the mixing in charged currents is given by the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [2] and the source of all CP violation is the unique phase in this matrix. The fact that some of the CKM matrix elements, especially the ones involving the top, are poorly known at present, implies that the unitarity of the 3×3 CKM matrix has yet to be tested. Moreover, little is known so far about possible FCNC for the top. Fortunately the situation is expected to improve in the near future. Many ongoing and future experiments (Tevatron, B factories, LHC) will be able to test the unitarity triangle [3] and to measure top couplings to within $\sim 1\%$ of the typical size of a weak coupling, thus improving present bounds by more than one order of magnitude [4]. The usual procedure to analyse these experiments is to assume the SM and determine the corresponding parameters from the measurements. New physics would then manifest itself as inconsistencies arising from the fact that the experiments overconstrain this parametrisation. For instance, the non-closure of the unitarity triangle would directly indicate non-standard physics. Obviously, this method is insufficient to learn about the new physics if a new effect is found. Moreover, a naïve use of the SM ansatz can be misleading when interpreting results hinting a closed unitarity triangle. In order to analyse and discuss new physics it seems more convenient to work with a more general parametrisation from the very beginning. A convenient parametrisation should reflect which parameters are small and vanish in the SM limit. Furthermore, one would like to know what relations between neutral and charged currents can be imposed and what bounds should be expected on general grounds (symmetry, dimensional analysis, etc.). All these issues can be best dealt with using effective theory techniques, and this is what we do in this paper. We also argue that only models with extra vector-like quarks can give large new effects beyond the SM.

The SM should be understood as the lowest dimension part in the expansion of an effective Lagrangian that describes any physics below a certain scale Λ (see [5] and references therein). This effective Lagrangian can describe a large class of SM extensions, including models with extra gauge interactions, new vector bosons, fermions or scalars with or without supersymmetry, and in four or higher dimensions [6,7]. Λ is a characteristic scale of the high energy theory, typically given by the lowest thresholds of the non-standard particles. An effective Lagrangian can describe both decoupling and non-decoupling non-standard physics (in the second case the effective description is mandatory). Here we shall assume that the low energy scalar sector is given by an elementary (not too heavy) Higgs and therefore work in a decoupling scenario [1]. Moreover, it is sufficient to consider only the SM Higgs for the following reasons: Scalar singlets which can acquire a vacuum expectation value (vev) do not transform under the SM and are then flavour blind. Other multiplets like triplets can only get very small vevs v_t (due to the constraints on the ρ parameter) and their effects are suppressed by powers of $\frac{v_t}{v}$, where v is the SM vev. Finally, for any number of Higgs doublets we identify the minimal SM one with the combination getting a vev.

The experimental precision and the scale Λ determine the order to be considered in the expansion

$$\mathcal{L}^{\mathit{eff}} = \mathcal{L}_4 + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \ldots \quad . \tag{1}$$

$\mathcal{L}_4$ is the SM Lagrangian and $\mathcal{L}_{5,6,\ldots}$ contain operators $\mathcal{O}_x$ of dimension 5, 6,. . ., respec-

[1] The same analysis of this paper could be repeated starting with the chiral SM [8]. The results should be analogous since the flavour structure is not altered.

tively. All these operators are invariant under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. There is an extensive literature on the operators in $\mathcal{L}_5$ and $\mathcal{L}_6$ and on their experimental constraints. In Ref. [6] it is shown that, assuming lepton and baryon number conservation, no dimension 5 operator can be constructed with the SM fields, while there are 80 independent gauge invariant operators of dimension 6 (up to flavour indices) included in

$$\mathcal{L}_6 = \alpha_x \mathcal{O}_x + h.c.\,. \tag{2}$$

The coefficients α_x parametrise the new physics beyond the SM. Other operators are allowed, but can be transformed into the ones in [6] by using the equations of motion of $\mathcal{L}_4$. This does not change S matrix elements to order $1/\Lambda^2$ [7]. It should be noted that the Lagrangian $\mathcal{L}^{\mathit{eff}}$ can be used beyond the classical level. Quantum corrections can be computed systematically as $\mathcal{L}^{\mathit{eff}}$ is renormalizable in the modern sense that there is a counterterm available to cancel every infinity [9]. A good and consistent approximation is to consider the full one-loop corrections to $\mathcal{L}_4$, but work at tree level whenever an operator in $\mathcal{L}_6$ is inserted. The effect of the SM radiative corrections should be taken into account when applying our results to precision data.

In order to compare with experiment we have to consider the spontaneous symmetry breaking (SSB) of the electroweak gauge symmetry, which introduces a new dimensionful parameter: the electroweak vev $v \sim 250$ GeV. Then $\mathcal{L}_6$ gives contributions $v^2\mathcal{L}'_4$, $v\mathcal{L}'_5$ and $\mathcal{L}'_6$, where $\mathcal{L}'_d$ contains operators of dimension d invariant under the unbroken $SU(3)_C \times U(1)_Q$ gauge symmetry. Quark mixing can occur in two-fermion and four-fermion operators. The latter have dimension 6 and the former (after SSB), dimension 4, 5 and 6. Operators with the same fields but with a different dimension after SSB can in principle be distinguished experimentally, since they lead to a different momentum dependence. Four-fermion operators can give non-standard mixing in kaon or B meson experiments, quark-pair production, etc. They are generated in many extensions of the SM. Here, however, we shall concentrate on the trilinear couplings $V\bar{q}q'$, $V = Z, W^\pm$, and $H\bar{q}q'$, which generalize the CKM matrix and the diagonal couplings in neutral currents. These vertices can be measured independently and may have a different origin than the four-fermion operators.

The operators $\mathcal{O}_x$ are generated by virtual exchange of the heavy modes in the high-energy theory. It is useful to distinguish the operators which are generated at tree level from those which are generated only by loop diagrams [7]. The latter are suppressed by powers of $1/16\pi^2$, at least when the heavy theory is weakly interacting. We can then consider only operators generated at tree level, since we are mainly interested in the largest SM deviations, which might be observed within the precision of future experiments. This restriction reduces the list of operators relevant to trilinear couplings to the seven operators collected in Table 1. These operators give only, after SSB, dimension 4 (i.e., $\sim v^2/\Lambda^2$) operators and operators of dimension 5 of the form $\partial H\, \bar{q}q'$. Hence, magnetic-moment type operators and other operators with extra momentum dependence are not generated at tree level. On the other hand, it is important to observe that these seven operators are the only ones contributing to the quark sector of $\mathcal{L}'_4$ [6]. Therefore, even though we are mainly concerned with large effects in the top couplings, all the results in this paper for the couplings $V\bar{q}q'$, $H\bar{q}q'$ apply to any kind of new physics, independently of whether it contributes at tree level or not. There are also other operators which would redefine the Z, $W^\pm$ and H fields in the trilinear quark couplings, but they are flavour blind and need not be taken into account for our purposes. No flavour change occurs in the dimension 4 couplings to the photon and gluons due to the exact $U(1)_Q$ and $SU(3)_C$ symmetry.

Table 1
Dimension 6 operators correcting trilinear $V\bar{q}q'$, $V = W^\pm, Z$, and $H\bar{q}q'$ vertices. (See Ref. [6] for notation.)

$$
\begin{aligned}
(\mathcal{O}_{\phi q}^{(1)})^{ij} &= (\phi^\dagger i D_\mu \phi)(\bar{q}_L^i \gamma^\mu q_L^j) \\
(\mathcal{O}_{\phi q}^{(3)})^{ij} &= (\phi^\dagger \tau^I i D_\mu \phi)(\bar{q}_L^i \gamma^\mu \tau^I q_L^j) \\
(\mathcal{O}_{\phi u})^{ij} &= (\phi^\dagger i D_\mu \phi)(\bar{u}_R^i \gamma^\mu u_R^j) \\
(\mathcal{O}_{\phi d})^{ij} &= (\phi^\dagger i D_\mu \phi)(\bar{d}_R^i \gamma^\mu d_R^j) \\
(\mathcal{O}_{\phi \phi})^{ij} &= (\phi^T \epsilon i D_\mu \phi)(\bar{u}_R^i \gamma^\mu d_R^j) \\
(\mathcal{O}_{u \phi})^{ij} &= (\phi^\dagger \phi)(\bar{q}_L^i \tilde{\phi} u_R^j) \\
(\mathcal{O}_{d \phi})^{ij} &= (\phi^\dagger \phi)(\bar{q}_L^i \phi d_R^j)
\end{aligned}
$$

In the following we extend the analysis of Ref. [6] to describe quark mixing. We generalize the CKM matrix and the GIM mechanism, and find relations and bounds fulfilled by the trilinear couplings. As an illustration of how the general description can be employed to study particular models, we discuss a simple extension of the SM with an extra exotic up quark isosinglet T [10,11]. This example also shows that a top mixing large enough to be observable at future colliders can actually be produced in explicit models [12]. Finally, we argue that the largest possible departure from the SM is obtained in models with heavy vector-like fermions. These models are analised in a subsequent paper [11].

The seven operators in Table 1, together with Eqs. (1) and (2), describe the large contributions of an arbitrary SM extension to the trilinear quark couplings, and any contribution (large or small) to non-derivative trilinear quark couplings. Note that the actual value of the coefficients α_x depends on the specific basis one uses for the quark fields, which is not completely fixed by the requirement of canonical kinetic terms and diagonal gauge terms before SSB. The physical results are independent of the choice of basis, but for definiteness we shall use here the basis in which the SM Yukawa couplings of the down quarks are diagonal, real and positive and the SM Yukawa couplings of the up quarks are of the form $\lambda_{ij}^u = V_{ij}^\dagger \lambda_j^u$ with λ_i^u real and positive and V unitary (and identical to the CKM matrix in the SM). After SSB, the quark mass matrices to order v^2/Λ^2 can be made diagonal, real and positive by biunitary transformations $u_{L,R} = U_{L,R}^u \, u'_{L,R}$, $d_{L,R} = U_{L,R}^d \, d'_{L,R}$, where the prime is used for mass eigenstates. We omit it in the following, for all the subsequent expressions are written in the mass eigenstate basis. Up to the freedom to redefine the phases of the quark mass eigenstates, the diagonalizing matrices read (for non-degenerate masses, as in the case of the SM)

$$
\begin{aligned}
U_L^u &= V^\dagger(\mathbf{1} + \tfrac{v^2}{\Lambda^2}A_L^u)\,, \quad & U_R^u &= \mathbf{1} + \tfrac{v^2}{\Lambda^2}A_R^u\,, \\
U_L^d &= \mathbf{1} + \tfrac{v^2}{\Lambda^2}A_L^d\,, & U_R^d &= \mathbf{1} + \tfrac{v^2}{\Lambda^2}A_R^d\,,
\end{aligned}
\tag{3}
$$

where $\mathbf{1}$ is the identity matrix and $A_{L,R}^{u,d}$ are antihermitian matrices given by

$$
\begin{aligned}
(A_L^u)_{ij} &= \frac{1}{2}(1-\frac{1}{2}\delta_{ij})\frac{\lambda_i^u(V\alpha_{u\phi})^\dagger_{ij} + (-1)^{\delta_{ij}}(V\alpha_{u\phi})_{ij}\lambda_j^u}{(\lambda_i^u)^2-(-1)^{\delta_{ij}}(\lambda_j^u)^2}\,, \\
(A_R^u)_{ij} &= \frac{1}{2}(1-\frac{1}{2}\delta_{ij})\frac{\lambda_i^u(V\alpha_{u\phi})_{ij} + (-1)^{\delta_{ij}}(V\alpha_{u\phi})^\dagger_{ij}\lambda_j^u}{(\lambda_i^u)^2-(-1)^{\delta_{ij}}(\lambda_j^u)^2}\,, \\
(A_L^d)_{ij} &= \frac{1}{2}(1-\frac{1}{2}\delta_{ij})\frac{\lambda_i^d(\alpha_{d\phi})^\dagger_{ij} + (-1)^{\delta_{ij}}(\alpha_{d\phi})_{ij}\lambda_j^d}{(\lambda_i^d)^2-(-1)^{\delta_{ij}}(\lambda_j^d)^2}\,, \\
(A_R^d)_{ij} &= \frac{1}{2}(1-\frac{1}{2}\delta_{ij})\frac{\lambda_i^d(\alpha_{d\phi})_{ij} + (-1)^{\delta_{ij}}(\alpha_{d\phi})^\dagger_{ij}\lambda_j^d}{(\lambda_i^d)^2-(-1)^{\delta_{ij}}(\lambda_j^d)^2}\,.
\end{aligned}
\tag{4}
$$

Our initial choice of basis implies that only U_L^u is non-trivial at order 1. $\alpha_{u\phi}$ and $\alpha_{d\phi}$ are the coefficients of the operators that contribute to the quark masses (see Table 1). In terms of mass eigenstates, the generic quark couplings have the following form:

$$
\begin{aligned}
\mathcal{L}^Z &= -\frac{g}{2\cos\theta_W}\Big(\bar{u}_L^i X_{ij}^{uL}\gamma^\mu u_L^j + \bar{u}_R^i X_{ij}^{uR}\gamma^\mu u_R^j \\
&\quad -\bar{d}_L^i X_{ij}^{dL}\gamma^\mu d_L^j - \bar{d}_R^i X_{ij}^{dR}\gamma^\mu d_R^j - 2\sin^2\theta_W J^\mu_{\mathrm{EM}}\Big) Z_\mu, \\
\mathcal{L}^W &= -\frac{g}{\sqrt{2}}(\bar{u}_L^i W_{ij}^L\gamma^\mu d_L^j + \bar{u}_R^i W_{ij}^R\gamma^\mu d_R^j)W_\mu^+ + h.c., \\
\mathcal{L}^H &= -\frac{1}{\sqrt{2}}(\bar{u}_L^i Y_{ij}^u u_R^j + \bar{d}_L^i Y_{ij}^d d_R^j + h.c.)H \\
&\quad + \Big(\bar{u}_L^i Z_{ij}^{uL}\gamma^\mu u_L^j + \bar{u}_R^i Z_{ij}^{uR}\gamma^\mu u_R^j - \bar{d}_L^i Z_{ij}^{dL}\gamma^\mu d_L^j - \bar{d}_R^i Z_{ij}^{dR}\gamma^\mu d_R^j\Big)\, i\partial_\mu H,
\end{aligned}
\tag{5}
$$

The unbroken $U(1)_Q$ protects the terms proportional to J^μ_{EM}. The expressions to order $1/\Lambda^2$ of the coupling matrices X, W, Y and Z in terms of the coefficients α_x are:

$$
\begin{aligned}
X_{ij}^{uL} &= \delta_{ij} - \frac{1}{2}\frac{v^2}{\Lambda^2}V_{ik}(\alpha_{\phi q}^{(1)} + \alpha_{\phi q}^{(1)\dagger} - \alpha_{\phi q}^{(3)} - \alpha_{\phi q}^{(3)\dagger})_{kl}V^\dagger_{lj}, \\
X_{ij}^{uR} &= -\frac{1}{2}\frac{v^2}{\Lambda^2}(\alpha_{\phi u} + \alpha_{\phi u}^\dagger)_{ij}, \\
X_{ij}^{dL} &= \delta_{ij} + \frac{1}{2}\frac{v^2}{\Lambda^2}(\alpha_{\phi q}^{(1)} + \alpha_{\phi q}^{(1)\dagger} + \alpha_{\phi q}^{(3)} + \alpha_{\phi q}^{(3)\dagger})_{ij},
\end{aligned}
$$

$$
\begin{aligned}
X^{dR}_{ij} &= \frac{1}{2}\frac{v^2}{\Lambda^2}(\alpha_{\phi d}+\alpha^{\dagger}_{\phi d})_{ij}, \\
W^{L}_{ij} &= \tilde{V}_{ik}\left(\delta_{kj}+\frac{v^2}{\Lambda^2}(\alpha^{(3)}_{\phi q})_{kj}\right), \\
W^{R}_{ij} &= -\frac{1}{2}\frac{v^2}{\Lambda^2}(\alpha_{\phi\phi})_{ij}, \\
Y^{u}_{ij} &= \delta_{ij}\lambda^{u}_{j}-\frac{v^2}{\Lambda^2}\left(V_{ik}(\alpha_{u\phi})_{kj}+\frac{1}{4}\delta_{ij}[V_{ik}(\alpha_{u\phi})_{kj}+(\alpha^{\dagger}_{u\phi})_{ik}V^{\dagger}_{kj}]\right), \\
Y^{d}_{ij} &= \delta_{ij}\lambda^{d}_{j}-\frac{v^2}{\Lambda^2}\left((\alpha_{d\phi})_{ij}+\frac{1}{4}\delta_{ij}(\alpha_{d\phi}+\alpha^{\dagger}_{d\phi})_{ij}\right), \\
Z^{uL}_{ij} &= -\frac{1}{2}\frac{v}{\Lambda^2}V_{ik}(\alpha^{(1)}_{\phi q}-\alpha^{(1)\dagger}_{\phi q}-\alpha^{(3)}_{\phi q}+\alpha^{(3)\dagger}_{\phi q})_{kl}V^{\dagger}_{lj}, \\
Z^{uR}_{ij} &= -\frac{1}{2}\frac{v}{\Lambda^2}(\alpha_{\phi u}-\alpha^{\dagger}_{\phi u})_{ij}, \\
Z^{dL}_{ij} &= \frac{1}{2}\frac{v}{\Lambda^2}(\alpha^{(1)}_{\phi q}-\alpha^{(1)\dagger}_{\phi q}+\alpha^{(3)}_{\phi q}-\alpha^{(3)\dagger}_{\phi q})_{ij}, \\
Z^{dR}_{ij} &= \frac{1}{2}\frac{v}{\Lambda^2}(\alpha_{\phi d}-\alpha^{\dagger}_{\phi d})_{ij}.
\end{aligned} \tag{6}
$$

We have introduced the unitary matrix

$$\tilde{V} = V + \frac{v^2}{\Lambda^2}(VA^d_L - A^u_L V)\,. \tag{7}$$

Note that, to order $1/\Lambda^2$, we can substitute V by $\tilde{V}$ everywhere in Eq. (6), so that the different couplings depend on only one unitary matrix. These couplings incorporate features that are forbidden in the SM, namely, FCNC, right-handed neutral currents not proportional to J^{μ}_{EM}, right-handed charged currents, and left-handed charged currents which are not described by an unitary matrix. These effects are allowed in general to order $1/\Lambda^2$. We stress that these trilinear couplings can be directly determined from processes involving the $\bar{q}q'V$ and $\bar{q}q'H$ vertices in which the final V or H is observed. For the top quark this will be possible in large hadron colliders. Although the trilinear couplings also contribute to four-fermion processes (such as mixing of neutral mesons), one should remember that four-fermion operators may contribute in this case as well [10,13]. Nevertheless, one can still use these processes to put limits on the trilinear couplings under the assumption that no strong cancellations between cubic and quartic couplings take place.

The CKM matrix is now an arbitrary 3×3 matrix and can be written as the product of a unitary matrix V^L times a hermitian matrix H^L,

$$W^L = V^L H^L\,. \tag{8}$$

Alternatively one can write $W^L = H^{L'}V^L$, with $H^{L'} = V^L H^L V^{L\dagger}$. Basically, the decomposition in Eq. (8) is better suited to study the down sector, whereas the other one is more convenient for the up sector. For definiteness, we use the first decomposition in the following discussion. After using all the freedom to rotate the quark phases, W^L depends on 13 parameters. It is possible to express V^L in any of the existing parametrizations of the CKM matrix. V^L then depends on 3 angles and 1 phase and H^L depends on 6 real numbers and 3 phases. Since the SM deviations are small perturbations, H^L can be expanded around the identity. In our effective Lagrangian description,

$$H^L = \mathbf{1} + \frac{v^2}{2\Lambda^2} e^{-i\theta^d} \left(\alpha_{\phi q}^{(3)} + \alpha_{\phi q}^{(3)\,\dagger} \right) e^{i\theta^d} \,, \tag{9}$$

and $V^L = e^{-i\theta^u} \tilde{V} (\mathbf{1} + \frac{v^2}{2\Lambda^2} (\alpha_{\phi q}^{(3)} - \alpha_{\phi q}^{(3)\,\dagger}) e^{i\theta^d}$, where $e^{i\theta^{u,d}}$ are the diagonal phase matrices that bring V^L to the desired form. On the other hand, V^L can be expanded in the Cabibbo angle λ *à la* Wolfenstein [14] [2] . To order λ^3, the generalized CKM matrix reads

$$W^L = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} 1 + \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{12}^* & 1 + \omega_{22} & \omega_{23} \\ \omega_{13}^* & \omega_{23}^* & 1 + \omega_{33} \end{pmatrix} , \tag{10}$$

where the diagonal elements of H^L, ω_{ii}, are small real numbers and the off-diagonal ones, ω_{ij} for $i \neq j$, are small complex numbers. The parameters ω_{ij} vanish in the SM (they are $\sim v^2/\Lambda^2$). The ω_{ii} and the $|\omega_{ij}|$, $i \neq j$, are invariant under redefinitions of the quark phases, and depend on the hermitian part of the coefficient of a single operator, as shown in Eq. (9). The unitarity relations are modified at order $\sim v^2/\Lambda^2$. The non-closure of the unitarity triangles is directly given by the off-diagonal ω_{ij}. For example, we have

$$W_{ud} W_{ub}^* + W_{cd} W_{cb}^* + W_{td} W_{tb}^* = 2\omega_{13}^* \,. \tag{11}$$

The remaining ω_{ij} can be determined, in principle, from measurements of the other unitarity relations. Observe that the different "triangles" are not equivalent for a non-unitary CKM matrix. On the other hand, there are now four independent physical phases that can induce CP violation. In the alternative decomposition $W^L = H^{L'} V^L$, $H^{L'}$ is analogously parametrised by $\omega'_{ij} = V^L_{ik} \omega_{kl} V^{L\dagger}_{lj}$, where $\omega^{(\prime)}_{ij} = \omega^{(\prime)*}_{ji}$ for $i > j$. Note that $\omega'_{ij} = \omega_{ij}$ up to terms suppressed by the Cabibbo angle.

The mixing matrix for right-handed charged currents can be similarly decomposed as $W^R = V^R H^R$, where H^R is of order v^2/Λ^2. Note, however, that after bringing V^L to a

[2] It should be noted that the Wolfenstein parametrization introduces an artificial loss of unitarity which should not be confused with the physical loss of unitarity contained in H^L. This effect can be made arbitrarily small by considering sufficiently high orders in λ. For many purposes, like CP violation in neutral kaon decays or high precision CP studies of neutral B-meson decays, one should incorporate corrections proportional to λ^4 and λ^5.

given form there is no freedom to redefine V^R while keeping the masses real and positive. Therefore W^R has 18 independent real (and small) parameters.

The couplings to the massive vector bosons, X_{ij} and W_{ij} in Eq. (6), are well-known for the 5 lightest quarks and will be precisely measured at future colliders for the top quark. A relevant question is what can be said about the SM deviations on general grounds. There are 6 types of couplings X_{ij}, W_{ij} in Eq. (6) and they are a function of 5 types of operators and a unitary matrix. Combining the different expressions we find the general relations

$$
\begin{aligned}
X_{ij}^{uL} &= 2W_{ik}^{L}W_{kj}^{L\dagger} - W_{ik}^{L}X_{kl}^{dL}W_{lj}^{L\dagger}, && (12)\\
X_{ij}^{dL} &= 2W_{ik}^{L\dagger}W_{kj}^{L} - W_{ik}^{L\dagger}X_{kl}^{uL}W_{lj}^{L}. && (13)
\end{aligned}
$$

Other relations may be fulfilled by the couplings to the Z and $W^\pm$ for particular classes of models. A possibility is that the new physics is such that

$$\alpha_{\phi q}^{(1)} = a\alpha_{\phi q}^{(3)}, \tag{14}$$

with a a number. This occurs whenever all the heavy fields contributing at tree level have the same statistics and transform in the same representation of the gauge group. Then a is given by Clebsh-Gordan coefficients. This is the case of models with heavy exotic quarks of just one type [11]. If Eq. (14) holds then there are two additional relations:

$$
\begin{aligned}
X_{ij}^{uL} &= 2\frac{1+a}{3+a}\delta_{ij} + \frac{1-a}{3+a}W_{ik}^{L}X_{kl}^{dL}W_{lj}^{L\dagger}, && (15)\\
X_{ij}^{dL} &= 2\frac{1-a}{3-a}\delta_{ij} + \frac{1+a}{3-a}W_{ik}^{L\dagger}X_{kl}^{uL}W_{lj}^{L}, && (16)
\end{aligned}
$$

which substituted in Eq. (12) give

$$
\begin{aligned}
X_{ij}^{uL} &= \frac{1+a}{2}\delta_{ij} + \frac{1-a}{2}W_{ik}^{L}W_{kj}^{L\dagger}, && (17)\\
X_{ij}^{dL} &= \frac{1-a}{2}\delta_{ij} + \frac{1+a}{2}W_{ik}^{L\dagger}W_{kj}^{L}. && (18)
\end{aligned}
$$

These relations become particularly simple for $a = \pm 1$. In terms of the parametrisation of W^L in Eqs. (8) and (10), they read

$$
\begin{aligned}
X_{ij}^{uL} &= \delta_{ij} + (1-a)\omega'_{ij}, && (19)\\
X_{ij}^{dL} &= \delta_{ij} + (1+a)\omega_{ij}. && (20)
\end{aligned}
$$

The severe constraints on the couplings of the first five flavours to the Z imply that ω_{ij} are very small unless $a = -1$, especially for $i \neq j$ (and ω'_{ij} for $i, j \neq 3$ are very small unless $a = 1$) [10,12,15] [3]. Indeed, we have

$$
\begin{aligned}
&|\omega_{12}| \lesssim 4.1 \times 10^{-5}\,, \qquad |\omega_{13}| \lesssim 1.1 \times 10^{-3}\,, \qquad |\omega_{23}| \lesssim 1.9 \times 10^{-3}\,, \\
&|\omega'_{12}| \lesssim 1.2 \times 10^{-3}\,.
\end{aligned} \tag{21}
$$

Since for each i, j, ω'_{ij} and ω_{ij} are of the same order of magnitude, we can conclude that for models satisfying Eq. (14) and $a \neq -1$ the top flavour changing couplings X^{uL}_{it}, $i \neq t$ must be very small too, and will not be observed in the next generation of accelerators. Another possibility that leads to simple relations is

$$
\alpha^{(3)}_{\phi q} = 0\,. \tag{22}
$$

Then we have

$$
W^L_{ij} = \tilde{V}_{ij}, \tag{23}
$$

$$
X^{uL}_{ij} = 2\delta_{ij} - W^L_{ik} X^{dL}_{kl} W^{L\dagger}_{lj}\,. \tag{24}
$$

On the other hand, the couplings to the Higgs boson are completely arbitrary. In the SM there are no derivative couplings of two quarks and the Higgs and the non-derivative couplings are diagonal and proportional to the masses of the quarks. At order $1/\Lambda^2$, however, FCNC mediated by the Higgs are allowed at tree level, and (non-diagonal) derivative couplings to the Higgs may exist. These derivative couplings have the same form as the corrections to the couplings to the Z, but they involve the antihermitian part of the coefficients α_x, rather than the hermitian one. Hence such couplings cannot be generated in models that only give hermitian contributions.

Besides these relations, the couplings to the Z, $W^\pm$ and H also satisfy generic inequalities. Let us first discuss the bounds on the couplings of quarks to the Z. The coupling matrices X_{ij} are always hermitian, so they can be diagonalized by a unitary matrix. Let $x_{\max}$ and $x_{\min}$ be the maximum and the minimum of the eigenvalues of any X. These eigenvalues, and hence $x_{\max}$ and $x_{\min}$, are determined by the coefficients α_x entering the corresponding expressions in Eq. (6), and are close to 1 (0) for X^L and (X^R). Since $X_{ij} - x_{\min}\delta_{ij}$ and $x_{\max}\delta_{ij} - X_{ij}$ are positive semidefinite, the following positivity constraints are fulfilled:

$$
|X_{ij} - x_{\min}\delta_{ij}|^2 \leq (X_{ii} - x_{\min})(X_{jj} - x_{\min})\,, \tag{25}
$$

$$
|X_{ij} - x_{\max}\delta_{ij}|^2 \leq (x_{\max} - X_{ii})(x_{\max} - X_{jj})\,. \tag{26}
$$

[3] Here and in the limits in Eq. (31) below we are using the values in [15] for neutral currents of the quarks of the first family. These values give directly the corresponding couplings to the Z assuming that there are no significant contributions of non-standard four-fermion operators to atomic parity violation experiments.

If all the eigenvalues of a certain X^L are ≥ 1 (≤ 1), we can change $x_{\min}$ ($x_{\max}$) in Eq. (25) (Eq. (26)) by 1 and obtain an interesting bound which is independent of Λ and of the details of the α_x:

$$|X^L_{ij}|^2 \leq (X^L_{ii} - 1)(X^L_{jj} - 1), \text{ for } i \neq j\,. \tag{27}$$

Moreover, each diagonal element then fulfils,

$$X^L_{ii} \geq 1 \ (\leq 1). \tag{28}$$

Correspondingly, if all the eigenvalues of X^R are positive (negative) semidefinite, we obtain

$$|X^R_{ij}|^2 \ \leq \ X^R_{ii} X^R_{jj}\,, \tag{29}$$
$$X^R_{ii} \geq 0 \ \ (\leq 0). \tag{30}$$

Equation (29) is trivially satisfied by X^L but in that case it gives no phenomenologically interesting information. Eqs. (27) and (29), on the other hand, provide stringent constraints. Indeed, inserting the atomic parity violation and LEP data [15] for $X^{uL,R}_{uu,cc}$, $X^{dL,R}_{dd,bb}$, in Eqs. (27) and (29) we find the bounds

$$\begin{aligned} &|X^{uL}_{ut}| \leq 0.28,\ |X^{uR}_{ut}| \leq 0.14, \\ &|X^{uL}_{ct}| \leq 0.14,\ |X^{uR}_{ct}| \leq 0.16, \end{aligned} \tag{31}$$

at 90% C.L. (we have used the analogous analysis of [12]). These limits improve the present production bounds [4]. They must be fulfilled, for instance, in theories with one type of heavy exotic quark [11,12]. Even more stringent limits can be obtained from charged current data for models fulfilling Eq. (14). For example, if $a = -1$ and Eqs. (27) and (28) with "$\leq$" hold, we can use the measured values of the charged current couplings W^L_{ij} [12,15] (assuming the absence of right-handed charged currents) and Eq. (17) to obtain

$$\begin{aligned} &|X^{uL}_{ut}| \leq 0.05\,, \\ &|X^{uL}_{ct}| \leq 0.08\,. \end{aligned} \tag{32}$$

If the actual values are close to the maximum allowed by the bounds, the corresponding couplings will be observed at future colliders [4]. Such values can be reached in explicit models, as the one considered below.

Similar bounds can be obtained for the charged currents. The hermitian matrices $W^\dagger W = H^2$ are diagonalized by unitary transformations, which leads to inequalities for $W^{L,R\dagger} W^{L,R}$ depending only on the eigenvalues of $H^{L,R}$. (The inequalities for $W^{L,R} W^{L,R\dagger}$ are identical since both products give rise to equivalent matrices.) The equivalent of Eqs. (25) and (26) are obtained by changing X by $W^\dagger W$ and $x_{\rm max,min}$ by $h^2_{\rm max,min}$, where the latter are the maximum and minimum of H^2. If all the eigenvalues of H^L are ≥ 1 (≤ 1) we also have

$$
\begin{aligned}
|\sum_k W^{L*}_{ki} W^L_{kj}|^2 &\leq (\sum_k |W^L_{ki}|^2 - 1)(\sum_k |W^L_{kj}|^2 - 1)\,,\ \text{for } i \neq j\,, && (33)\\
\sum_k |W^L_{ki}|^2 &\geq 1\ (\leq 1)\,, && (34)
\end{aligned}
$$

where for clarity we have explicitly indicated the sum over k. Observe that the above condition is satisfied if and only if $\alpha^{(3)}_{\phi q} + \alpha^{(3)\dagger}_{\phi q}$ is positive (negative) semidefinite, and this depends only on general features of the high-energy theory. We show in [11] that extensions of the SM with heavy vector-like isotriplets (isosinglets) satisfy Eq. (34) with "$\geq$" ("$\leq$"). The constraints on W^R are trivially satisfied because the matrix $W^{R\dagger} W^R$ is always positive semidefinite. Bounds for the couplings to the Higgs can be obtained in a similar way, but we do not write them here.

The general analysis we have carried out can be applied to particular models, just by integrating the heavy modes out. As an example we consider now the addition of a heavy vector-like quark isosinglet of charge $\frac{2}{3}$, T. The full Lagrangian reads

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}^{SM} + \mathcal{L}_h + \mathcal{L}_{lh},\\
\mathcal{L}^{SM}_l &= \bar{q}^i_L i \not{D} q^i_L + \bar{u}^i_R i \not{D} u^i_R + \bar{d}^i_R i \not{D} d^i_R - (V^\dagger_{ij} \lambda^u_j \bar{q}^i_L \tilde{\phi}\, u^j_R + \lambda^d_i \bar{q}^i_L \phi\, d^i_R + \text{h.c.}) + \ldots, \qquad (35)\\
\mathcal{L}_h &= \bar{T}_L i \not{D} T_L + \bar{T}_R i \not{D} T_R - M(\bar{T}_L T_R + \bar{T}_R T_L),\\
\mathcal{L}_{lh} &= -\lambda'_j V_{ji} \bar{T}_R \tilde{\phi}^\dagger q^i_L + h.c.\,,
\end{aligned}
$$

where the dots stand for terms not involving the quarks. In this case, $\Lambda = M$, the mass of the exotic quark. The integration of the field T gives (see Ref. [11] for more details)

$$
\begin{aligned}
\mathcal{L}_4 &= \mathcal{L}^{SM}, && (36)\\
\frac{1}{\Lambda^2}\mathcal{L}_6 &= V^\dagger_{ik} \lambda'^*_k \lambda'_l V_{lj}\, \bar{q}^i_L \tilde{\phi} \frac{i\not{D}}{M^2} (\tilde{\phi}^\dagger q^j_L).
\end{aligned}
$$

We want to express this result in the operator basis $\mathcal{O}_x$ in Table 1, which requires the use of the equations of motion of $\mathcal{L}_4$. We find

$$
\begin{aligned}
(\alpha^{(1)}_{\phi q})_{ij} &= \frac{1}{4} V^\dagger_{ik} \lambda'^*_k \lambda'_l V_{lj},\\
(\alpha^{(3)}_{\phi q})_{ij} &= -(\alpha^{(1)}_{\phi q})_{ij}, && (37)\\
(\alpha_{u\phi})_{ij} &= \frac{1}{2} V^\dagger_{ik} \lambda'^*_k \lambda'_j \lambda^u_j.
\end{aligned}
$$

The other coefficients vanish. Now we just have to substitute these coefficients in Eq. (6) to obtain

$$
\begin{aligned}
X_{ij}^{uL} &= \delta_{ij} - \frac{1}{2}\frac{v^2}{M^2}\lambda_i'^*\lambda_j', \\
X_{ij}^{uR} &= 0, \quad X_{ij}^{dL} = \delta_{ij}, \quad X_{ij}^{dR} = 0, \\
W_{ij}^{L} &= \left(\delta_{ik} - \frac{1}{4}\frac{v^2}{M^2}\lambda_i'^*\lambda_k'\right)\tilde{V}_{kj}, \\
W_{ij}^{R} &= 0, \\
Y_{ij}^{u} &= \left(\delta_{ij} - \frac{1}{2}\frac{v^2}{M^2}(1+\frac{1}{2}\delta_{ij})\lambda_i'^*\lambda_j'\right)\lambda_j^u, \\
Y_{ij}^{d} &= \delta_{ij}\lambda_j^d, \\
Z_{ij}^{uL} &= Z_{ij}^{uR} = Z_{ij}^{dL} = Z_{ij}^{dR} = 0.
\end{aligned}
\tag{38}
$$

The decomposition of W^L in Eq. (8) holds for $V^L = \tilde{V}$, $H^L = \mathbf{1} - \frac{1}{4}\frac{v^2}{M^2}\tilde{V}^\dagger(\lambda'^*\lambda')\tilde{V}$ and $H^{L'} = \mathbf{1} - \frac{1}{4}\frac{v^2}{M^2}(\lambda'^*\lambda')$, where the initial V is such that no further phase redefinitions are necessary. The coefficients in Eq. (37) satisfy the relation (14) with $a = -1$ and $\alpha_{\phi q}^{(3)}$ is negative semidefinite. Therefore, this model fulfils Eqs. (15–20) with $a = -1$, and Eqs. (27–30,33,34) with "$\leq$". In particular, the limits (32) and (31) must be satisfied. Values of the Yukawas λ' can be found such that these limits are saturated. Basically, the maximal values are achieved when the new quark mixes with t and with either u or c, but not with both u and c. Otherwise the tight experimental bound on X_{uc}^{uL} would require a very large mass M, and hence a small X_{ut}^{uL} and X_{ct}^{uL}.

Finally, it is interesting to see what kinds of models can produce large new effects in quark mixing. Of course, one necessary condition is that the high energy scale Λ be sufficiently low ($\sim$ 1TeV). The other necessary condition (assuming weak coupling) is that the operators in $\mathcal{L}^{\mathit{eff}}$ be generated at tree level. A classification of the new physics that contributes at tree level to each process was given in Ref. [7]. If the high energy theory is a renormalizable gauge theory, the only heavy particles that generate at tree level the operators in Table 1 with a covariant derivative are either extra gauge bosons or extra quarks. Diagrams contributing to $\bar{q}q'V$ vertices are depicted in Fig. 1. The new vector bosons can mix with the Z and the W and hence generate trilinear couplings through the first tree-level diagram in Fig. 1 [16]. However, the mixing angle θ between the new and standard gauge bosons is constrained to be $\lesssim 0.01$ by the Z-pole data from LEP [15]. Hence, besides the suppression v^2/Λ^2 there is an additional suppression given by θ. Therefore the largest mixing effects arise in models with extra quarks. On the other hand, the mass of chiral quarks is given by their Yukawa couplings to the Higgs and must be $\sim v$. But the only light quarks allowed by precission electroweak studies are the standard ones. Hence, the extra quarks giving rise to large mixing must be vector-like, *i.e.*, their left-handed and right-handed components must transform in the same representation of the gauge group. These particles are present in many well-motivated SM

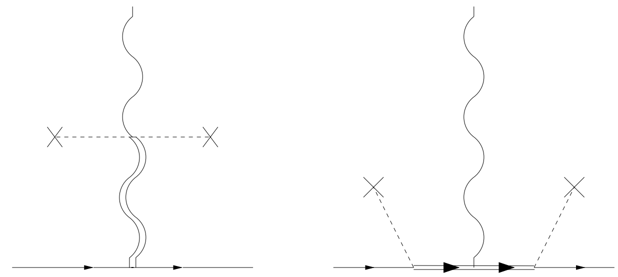

Fig. 1. Diagrams generating new $\bar{q}q'V$ couplings at tree level. The first diagram represents the exchange of a heavy vector boson that mixes with V and the second one represents the exchange of a heavy quark that mixes with q and q'.

extensions [10]. Besides, they must couple to the standard quarks, which restricts their possible representations: only singlets, doublets and triplets under $SU(2)_L$ contribute. Their hypercharges are also constrained. In Ref. [11] we present a detailed study of quark mixing in general models with exotic quarks above the electroweak scale. Analyses of quark mixing in particular models can be found in Ref. [10].

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